MATH 1000 - Quiz 2

Name: ____

Instructions: Write cleanly, show all work. Explain any trick questions.

1. Expand the following:

   (a) $(3+\sqrt{2})\cdot(2-\sqrt{2}) =$

Solution: Here it is all in one go:

$$\begin{aligned}(3+\sqrt{2})\cdot(2-\sqrt{2}) &= (3+\sqrt{2})\cdot 2 + (3+\sqrt{2})\cdot(-\sqrt{2})\\ &= (3+\sqrt{2})\cdot 2 - (3+\sqrt{2})\cdot\sqrt{2}\\ &= \left(3\cdot 2+\sqrt{2}\cdot 2\right) - \left(3\cdot\sqrt{2}+\sqrt{2}\cdot\sqrt{2}\right)\\ &= 3\cdot 2+\sqrt{2}\cdot 2 - 3\cdot\sqrt{2} - \sqrt{2}\cdot\sqrt{2}\\ &= 6+2\sqrt{2}-3\sqrt{2}-2\\ &= 4-\sqrt{2}\end{aligned}$$

Steps: The first step is the distributive law

$$a\cdot(b+c) = a\cdot b + a\cdot c$$

Here I used $a=(3+\sqrt{2})$, $b=2$, and $c=-\sqrt{2}$. Be careful with the negative. So

$$(3+\sqrt{2})\cdot(2-\sqrt{2}) = (3+\sqrt{2})\cdot 2 + (3+\sqrt{2})\cdot(-\sqrt{2})$$

or

Here I put parentheses around $-\sqrt{2}$ when I substitute it in for c, to remember the I'm multiplying by a negative number, not subtracting.

Then it is probably easier to multiply the negative and make it a subtraction (of the entire product $(3+\sqrt{2})\cdot\sqrt{2}$, not just the $\sqrt{2}$ part).

$$(3+\sqrt{2})\cdot 2 + (3+\sqrt{2})\cdot(-\sqrt{2}) = (3+\sqrt{2})\cdot 2 - (3+\sqrt{2})\cdot\sqrt{2}$$

From the second to the third line I used the distributive property again, from the other side

$$(b+c)\cdot a = (b\cdot a + c\cdot a)$$

I put the parentheses on the right again to help me think of it as 'one thing'. In fact, I used it twice: first with $b=3$, $c=\sqrt{2}$, and $a=2$, and then a second time with $b=3$, $c=\sqrt{2}$, and $a=\sqrt{2}$. So

$$(3+\sqrt{2})\cdot 2 - (3+\sqrt{2})\cdot\sqrt{2} = \left(3\cdot 2+\sqrt{2}\cdot 2\right) - \left(3\cdot\sqrt{2}+\sqrt{2}\cdot\sqrt{2}\right)$$

Then I also distribute the negative sign across the parentheses:

$$\left(3 \cdot 2 + \sqrt{2} \cdot 2\right) - \left(3 \cdot \sqrt{2} + \sqrt{2} \cdot \sqrt{2}\right) = 3 \cdot 2 + \sqrt{2} \cdot 2 - 3 \cdot \sqrt{2} - \sqrt{2} \cdot \sqrt{2}$$

Here you see why I put the parentheses in, it is easy to forget and write $-3 \cdot \sqrt{2} + \sqrt{2}\sqrt{2}$.

After we've expanded everything, we do the multiplications and collect like terms. Here 'like terms' means the integer part and the part with a $\sqrt{2}$.

(b) $\left(\frac{9x^4}{16}\right)^{\frac{3}{2}} =$

Solution: Again, I'll give a full answer first:

$$\begin{aligned}\left(\frac{9x^4}{16}\right)^{\frac{3}{2}} &= \frac{(9x^4)^{\frac{3}{2}}}{16^{\frac{3}{2}}} \\ &= \frac{9^{\frac{3}{2}} \cdot (x^4)^{\frac{3}{2}}}{16^{\frac{3}{2}}} \\ &= \frac{3^3 \cdot (x^2)^3}{4^3} \\ &= \frac{27x^6}{64}\end{aligned}$$

All I use is the power law

$$(a \cdot b)^n = a^n \cdot b^n$$

Remember this works for fractions too:

$$\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}$$

(in fact these are the same, if $b = \frac{1}{c}$.

So, I put the power in the numerator and denominator separately, then on each part of the numerator separately, then try to evaluate. To get from the second to the third line, remember how fractional powers work: there's a whole number power and a radical, and you can do them in any order

$$9^{\frac{3}{2}} = \sqrt{9^3} = (\sqrt{9})^3$$

Since I recognized $\sqrt{9} = 3$, $\sqrt{x^4} = x^2$, and $\sqrt{16} = 4$, I did all the square roots first.

Finally do the multiplications. As a small shortcut, remember

$$(x^2)^3 = x^{2\cdot 3} = x^6$$

instead of writing out $x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$